Journal of Applied Mechanics and Technical Physics, Vol. 35, No. 1, 1994

CALCULATION OF ELASTOPLASTIC STRAINS IN A MULTICOMPONENT POLYCRYSTALLINE MATERIAL

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UDC 539.374

Study of the laws governing the elastoplastic deformation of composite materials obtained on the basis of several finely dispersed components continues to be of considerable interest in connection with their broadening use [1-3]. However, the methods presently available for studying the inelastic behavior of stochastic mixtures do not adequately account for the physical nature of plastic deformation in their components.

In this investigation, we calculate the plastic strains in a stochastic composite and derive its macroscopic flow equation for an arbitrary loading program. We do this on the basis of a mechanism which includes multiple crystallographic slip in the components of single crystals. A similar approach was used in [4] for a one-component polycrystal. A comparison of theoretical and experimental tension curves made for the example of a copper-tungsten composite showed that the theory and experiment agree satisfactorily.

1. We will examine a polycrystalline composite medium consisting of N uniformly mixed, ideally bonded homogeneous and elastic isotropic components. Let q = 1, ..., M represent the elastic components and q = M + 1, ..., N represent the elastoplastic components.

We will assume that the plastic strains in the grains (single crystals) of the components of the composite occur by translational crystallographic slip and begin at the moment when the shear stress in any slip system of a grain reaches a value $\tau_0^{(q)}$, which is a constant for the given material. The components in which this condition is not yet satisfied for the specified load history remain elastic. Plastic shears reinforce both the active and the passive slip systems in the single crystal, which is expressed in an increase in the resolvent shear stress.

Thus, the following relations should be satisfied for a plastic grain in the q-th (q = M + 1, ..., N) component of the medium

$$t_{ij}^{\alpha(q)} s_{ij}^{(q)} = \tau^{\alpha(q)}, \quad \varepsilon_{ij}^{(q)} = 2t_{ij}^{\alpha(q)}\lambda_\alpha,$$
$$t_{ij}^{\alpha(q)} = \frac{1}{2}\left[l_i^{(\alpha)(q)} n_j^{(\alpha)(q)} + l_j^{(\alpha)(q)} n_i^{(\alpha)(q)}\right] \quad (i, j = 1, 2, 3). \tag{1.1}$$

Here, $s_{ij}^{(q)}$ are the deviators of the local stresses $\sigma_{ij}^{(q)}$; $\varepsilon_{ij}^{(q)}$ are the local plastic strains ($\varepsilon_{ii}^{(q)} = 0$); $\tau^{\alpha(q)}$, λ_α are the shear stress and plastic shear in the system α of the grain; $t_{ij}^{\alpha(q)}$ is the Schmidt tensor, where $\mathbf{l}^{(\alpha)(q)}$, $\mathbf{n}^{(\alpha)(q)}$ are unit vectors for the slip direction and a normal to the slip plane of system α. No summation is performed over the indices.

We will assume that the reinforcement of the slip systems of the single crystals in all of the plastic components of the medium (q = M + 1, ..., N) is isotropic and is described by the equation [5]

$$\tau^{\alpha(q)} = \tau_0^{(q)} + \int H^{(q)} \sum_{\beta=1}^{n_q} d\lambda_\beta \quad (\alpha = 1, ..., L_q), \tag{1.2}$$

where $H^{(q)}$ is the isotropic strain-hardening modulus; $\tau_0^{(q)}$ is the resolvent shear stress associated with the initial shear, this stress depending on the elastic properties of the component and its microstructure [6, 7]; n_q is the number of active slip systems in

Moscow. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 1, pp. 121-128, January-February, 1994. Original article submitted June 25, 1992; revision submitted November 12, 1992.

0021-8944/94/3501-0123$12.50

the given plastic grain of the composite; L_q is the total number of slip systems in the crystal. It follows from Eq. (1.2) that the acting shear stresses are the same in all of the active systems of the single crystal.

In accordance with [8], we assume that the plastic shears in the slip systems of the crystal will be equal in the case of multiple slip. Then introducing the integral shear $\Lambda^{(q)} = n_q\lambda$ ($\lambda_\alpha = \lambda$, $\alpha = 1, \ldots, n_q$) and the Schmidt tensor $t_{ij}^{(q)}$, averaged over all active slip systems, we use (1.1) and (1.2) to obtain expressions for the increments of the local stresses and strains ($\dot{a} \equiv da$) in terms of $\dot{\Lambda}^{(q)}$:

$$t_{ij}^{(q)}\dot{s}_{ij}^{(q)} = H^{(q)}\dot{\Lambda}^{(q)}, \quad \dot{\varepsilon}_{ij}^{(q)} = 2t_{ij}^{(q)}\dot{\Lambda}^{(q)}$$
$$\left(t_{ij}^{(q)} = \frac{1}{n_q}\sum_{\alpha=1}^{n_q} t_{ij}^{\alpha(q)}\right). \tag{1.3}$$

Assuming that the strain-hardening of the components is regular [9], we assign their local strain-hardening moduli in the form

$$H^{(q)} = 2h_{(q)}t_{kl}^{(q)}t_{kl}^{(q)} \tag{1.4}$$

(h_q are physical constants of the materials).

It was shown in [4] that the choice of $H^{(q)}$ in the form (1.4) models features of the regular strain-hardening of a material at the microstructural level.

2. We describe the spatial structure of the composite medium by means of random indicator functions of the coordinates $\varkappa_q(r)$ ($q = 1, \ldots, N$), taking a value of 1 for the set of points of the q-th component and 0 outside this set. Also, let c_q be the volume concentrations of the components. Using these functions, we can express Hooke's law (which is valid at the points of the component) in the form

$$s_{ij}(r) = 2\mu(r)\,[e_{ij}(r) - \varepsilon_{ij}(r)],$$
$$\sigma_{ii}(r) = 3K(r)\,E_{ii}(r),$$
$$\varepsilon_{ij}(r)\,\varkappa_q(r) \equiv 0 \quad (q = 1, \ldots, M), \tag{2.1}$$
$$\mu(r) = \sum_{q=1}^{N}\mu_q\varkappa_q(r), \quad K(r) = \sum_{q=1}^{N}K_q\varkappa_q(r),$$

where $E_{ij}(r)$ and $e_{ij}(r)$ is the tensor of the total local strains of the medium and its deviator; K_q and μ_q are the elastic moduli of the components of the composite ($q = 1, \ldots, N$).

We now assume that all of the random fields being examined

$$E_{ij}(r)\ \ (e_{ij}(r)), \quad \sigma_{ij}(r)\ \ (s_{ij}(r)), \quad \varepsilon_{ij}(r) \text{ and } \varkappa_q(r)$$

($q = 1, \ldots, N$) are statistically uniform and ergodic. Then their mathematical expectations can be replaced by quantities averaged over the volumes of the components V_q and the composite V. The corresponding averaging operations will be designated by the symbols $\langle\ \rangle_q$ and $\langle\ \rangle$.

Equilibrium equations $\nabla_j\sigma_{ij}(r) = 0$ (∇_j is the operator for differentiation with respect to the j-th coordinate) must also be satisfied at each point of the medium. With allowance for these equations, we write Eqs. (2.1) in the Lamé form and then use the Green's function of an equivalent infinite homogeneous medium to convert them to integral form. For simplicity limiting ourselves to the singular approximation hypothesis [10], we obtain local relations of the form

$$e_{ij}(r) = \langle e_{ij}\rangle + \alpha_0\,[Q_{ij}(r) - \langle Q_{ij}\rangle],$$
$$E_{ii}(r) = \langle E_{ii}\rangle + \beta_0 P(r). \tag{2.2}$$

Here

$$Q_{ij}(r) = e_{ij}(r) - \sum_{q=1}^{N} m_q\varkappa_q(r)\,[e_{ij}(r) - \varepsilon_{ij}(r)]; \tag{2.3}$$

$$P(\mathrm{r}) = E_{ii}(\mathrm{r}) - \sum_{q=1}^{N} k_q(\mathrm{r})\,\varkappa_q(\mathrm{r})\,E_{ii}(\mathrm{r});$$

$$\langle Q_{ij}\rangle = \mathscr{E}_{ij} = \langle s_{ij}\rangle (2\mu_0)^{-1} - \langle e_{ij}\rangle;$$
$$\alpha_0 = \frac{2(4-5\nu_0)}{15(1-\nu_0)}; \quad \beta_0 = \frac{1+\nu_0}{3(1-\nu_0)}; \quad \nu_0 = \frac{3K_0 - 2\mu_0}{3(\mu_0 + 3K_0)};$$
$$m_q = \mu_q\mu_0^{-1}; \quad k_q = K_qK_0^{-1};$$

$\mathscr{E}_{ij}$ are macroscopic plastic strains; ν_0 is the Poisson's ratio of the effective medium; μ_0 and K_0 are its macroscopic elastic constants. Thus,

$$\langle s_{ij}\rangle = 2\mu_0(\langle e_{ij}\rangle - \mathscr{E}_{ij}), \quad \langle\sigma_{ii}\rangle = 3K_0\langle E_{ii}\rangle. \tag{2.4}$$

We obtain the following from (2.1)-(2.3) for an arbitrary grain of the q-th component

$$\begin{gathered} e_{ij}^{(q)} = \langle e_{ij}\rangle + \alpha_0[\varepsilon_{ij}^{(q)} - m_q(e_{ij}^{(q)} - \varepsilon_{ij}^{(q)}) - \langle e_{ij}\rangle + \langle s_{ij}\rangle(2\mu_0^{-1})], \\ E_{ii}^{(q)} = \langle E_{ii}\rangle + \beta_0(1-k_q)E_{ii}^{(q)} \quad (q = 1, \ldots, N), \\ \varepsilon_{ij}^{(q)} \equiv 0 \quad (q = 1, \ldots, M). \end{gathered} \tag{2.5}$$

Solving (2.5) for $e_{ij}^{(q)}$ and $E_{ii}^{(q)}$, we use (2.1) to obtain the local stresses

$$\begin{gathered} s_{ij}^{(q)} = 2\mu_0 m_q^0[\eta_{ij} - (1-\alpha_0)\varepsilon_{ij}^{(q)}] \\ (\varepsilon_{ij}^{(q)} \equiv 0, \quad q = 1, \ldots, M), \\ \sigma_{ii}^{(q)} = 3K_0k_q^0\langle E_{ii}\rangle. \end{gathered} \tag{2.6}$$

Here,

$$\begin{gathered} \eta_{ij} = (1-\alpha_0)\langle e_{ij}\rangle + \alpha_0\langle s_{ij}\rangle(2\mu_0)^{-1}, \\ m_q^0 = m_q[1 + \alpha_0(m_q - 1)]^{-1}, \quad k_q^0 = k_q[1 + \beta_0(k_q - 1)]^{-1}. \end{gathered} \tag{2.7}$$

We further note that, in accordance with the chosen mechanism of plastic deformation, we can distinguish three different strain regions inside the q-th elastoplastic component of the composite. The first (s = 1) is formed of grains that are completely plastic, i.e. grains for which the number of active slip systems $n_q = p_q \leq 5$, where p_q is the maximum possible number of independent systems for the material of the composite. The second region (s = 2) is composed of crystallites that are incompletely plastic and have $n_q < p_q$ active slip systems. The third region (s = 3) comprises elastic grains, where plastic flow has not yet begun ($n_q = 0$). Let v_{q1}, v_{q2}, v_{q3} ($v_{q1} + v_{q2} + v_{q3} = 1$) be the volume concentrations of these single crystals in the q-th component of the mixture, and let $\boldsymbol{\varkappa}_{qs}(\mathbf{r})$ (s = 1, 2, 3) be random indicator functions of the corresponding strain regions inside V_q (q = M + 1, ..., N). It is evident that $\boldsymbol{\varkappa}_{q1}(\mathbf{r}) + \boldsymbol{\varkappa}_{q2}(\mathbf{r}) + \boldsymbol{\varkappa}_{q3}(\mathbf{r}) = \boldsymbol{\varkappa}_q(\mathbf{r})$ [$\boldsymbol{\varkappa}_q(\mathbf{r})$ will be the indicator function of the q-th elastoplastic component of the composite].

Using the notation introduced above, we can rewrite the first relation of (2.6) in the form

$$\begin{gathered} s_{ij}^{(q)}(\mathrm{r}) = 2\mu_0 m_q^0[\eta_{ij} - (1-\alpha_0)\varepsilon_{ij}^{(q)}(\mathrm{r})\,\varkappa_{q*}(\mathrm{r})] \\ (\varepsilon_{ij}^{(q)}(\mathrm{r})\,\varkappa_{q*}(\mathrm{r}) \equiv 0, \quad q = 1, \ldots, M), \end{gathered} \tag{2.8}$$

where $\boldsymbol{\varkappa}_{q*}(\mathbf{r}) = \boldsymbol{\varkappa}_{q1}(\mathbf{r}) + \boldsymbol{\varkappa}_{q2}(\mathbf{r})$ is the indicator function of the plastic region inside V_q.

Following [4], we will determine all of the local plastic strains in all of the strain regions of the q-th elastoplastic component. With allowance for (1.4), we obtain the following from (1.3) for a plastic grain in the volume V_q

$$t_{ij}^{(q)}[\dot{s}_{ij}^{(q)} - h_q\dot{\varepsilon}_{ij}^{(q)}] = 0. \tag{2.9}$$

In the completely plastic region (s = 1), Eq. (2.9) should be satisfied for all crystallographic directions due to the disappearance of strain anisotropy in the grains at this stage, i.e.,

$$\dot{s}_{ij(1)}^{(q)} - h_q \dot{\varepsilon}_{ij(1)}^{(q)} = 0. \tag{2.10}$$

We find from (2.10) and (2.8) that

$$\begin{gathered} \varepsilon_{ij(1)}^{(q)} = m_q^* \eta_{ij} \quad (q = M+1, \ldots, N), \\ m_q^* = m_q \left[(1-\alpha_0)(\vartheta_q + m_q) + \alpha_0 m_q \vartheta_q\right]^{-1}, \quad \vartheta_q = h_q (2\mu_0)^{-1}. \end{gathered} \tag{2.11}$$

For single crystals in the partially plastic region (s = 2), we use Eqs. (2.8-2.9) and (1.3-1.4) to determine the integral shear

$$\dot{\Lambda}_{(2)}^{(q)} = m_q^* \dot{\eta}_{ij} t_{ij}^{(q)} / 2 t_{pr}^{(q)} t_{pr}^{(q)} \quad (r, p = 1, 2, 3),$$

We then use (1.3) to obtain the corresponding plastic strains

$$\begin{gathered} \dot{\varepsilon}_{ij(2)}^{(q)} = m_q^* \psi_{ijkl}^{(q)} \dot{\eta}_{kl} \quad (q = M+1, \ldots, N), \\ \psi_{ijkl}^{(q)} = t_{ij}^{(q)} t_{kl}^{(q)} / t_{pr}^{(q)} t_{pr}^{(q)}. \end{gathered} \tag{2.12}$$

Finally, in the elastic region (s = 3), we find from (2.8) that

$$\dot{\varepsilon}_{ij(3)}^{(q)} \equiv 0, \quad \dot{s}_{ij(3)}^{(q)} = 2\mu_0 m_q^0 \dot{\eta}_{ij} \quad (q = M+1, \ldots, N). \tag{2.13}$$

To determine the macroscopic stresses in the composite, we average the local stresses over the volume of the specimen V with allowance for the rule on mechanical mixing of the components:

$$\langle s_{ij} \rangle = \sum_{q=1}^{N} c_q \langle s_{ij} \rangle_q, \quad \langle \sigma_{ii} \rangle = \sum_{q=1}^{N} c_q \langle \sigma_{ii} \rangle_q. \tag{2.14}$$

We find the quantities $\langle \dot{s}_{ij} \rangle_q$ by averaging (2.8) over the volume V_q with allowance for Eqs. (2.11)-(2.13):

$$\begin{gathered} \langle \dot{s}_{ij} \rangle_q = 2\mu_0 m_q^0 \{\dot{\eta}_{ij} - (1-\alpha_0)\left[v_{q1} \langle \dot{\varepsilon}_{ij} \rangle_{q1} + v_{q2} \langle \dot{\varepsilon}_{ij} \rangle_{q2}\right]\} \quad (q = 1, \ldots, N), \\ v_{q1} = v_{q2} \equiv 0 \quad (q = 1, \ldots, M). \end{gathered} \tag{2.15}$$

Here, $\langle \dot{\varepsilon}_{ij} \rangle_{q1}$ and $\langle \dot{\varepsilon}_{ij} \rangle_{q2}$ are the increments of the mean plastic strains in the completely and partially plastic regions of the q-th component of the composite.

It follows from Eqs. (2.11) that the plastic strains in the first strain region are constant, i.e. the equality $\dot{\varepsilon}_{ij(1)}^{(q)} \equiv \langle \dot{\varepsilon}_{ij} \rangle_{q1}$. The quantities $\dot{\varepsilon}_{ij(2)}^{(q)}$, determined by Eq. (2.12), depend on the orientation of the grains relative to the macroscopic axes of the specimen, and should be averaged over the region $\Omega \equiv \Omega_{q2}$ occupied by the orientations of crystallites of the second type in the orientation space of single crystals of the q-th component: $\langle \dot{\varepsilon}_{ij} \rangle_{q2} = \langle \dot{\varepsilon}_{ij(2)} \rangle_{\Omega}$.

With allowance for the above, we find from (2.15) that

$$\begin{gathered} \langle \dot{s}_{ij} \rangle_q = 2\mu_0 m_q^0 \{\left[1 - (1-\alpha_0) m_q^* v_{q1}\right] \dot{\eta}_{ij} - \\ - (1-\alpha_0) m_q^* v_{q2} \Psi_{ijkl}^{(q)} \dot{\eta}_{kl}\} \quad (q = 1, \ldots, N), \end{gathered}$$

where $\Psi_{ijkl}^{(q)} = \langle \psi_{ijkl}^{(q)} \rangle_{\Omega}$.

Inserting the expressions found for $\langle \dot{s}_{ij} \rangle_q$ into Eqs. (2.14), we determine the deviators of the macroscopic stresses in the medium:

$$\langle \dot{s}_{ij} \rangle = 2\mu_0 \{[1 - (1 - \alpha_0) B(\nu_1)] I_{ijkl} - (1 - \alpha_0) \Phi_{ijkl}(\nu_2)\} \dot{\eta}_{kl}. \tag{2.16}$$

Here

$$\nu_1 = (\upsilon_{(M+1)1}, \ldots, \upsilon_{N1}); \quad \nu_2 = (\upsilon_{(M+1)2}, \ldots, \upsilon_{N2});$$
$$B(\nu_1) = \sum_{q=M+1}^{N} c_q \upsilon_{q1} m_q^0 m_q^*; \quad I_{ijkl} = \frac{1}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk});$$
$$\Phi_{ijkl}(\nu_2) = \sum_{q=M+1}^{N} c_q \upsilon_{q2} m_q^0 m_q^* \Psi_{ijkl}^{(q)};$$

where δ_{ij} is the Kronecker tensor.

In Eqs. (2.16), the right sides were transformed with allowance for relations used in the self-consistent averaging scheme in [10]

$$\sum_{q=1}^{N} c_q m_q^0 = 1, \quad \sum_{q=1}^{N} c_q k_q^0 = 1,$$

which is used to determine the macroscopic elastic constants μ_0 and K_0 introduced by Eqs. (2.4).

Inserting $\dot{\eta}_{kl}$ from (2.7) into Eqs. (2.16) and solving the latter for $\langle \dot{s}_{ij} \rangle$, we obtain the macroscopic rheological equation of the elastoplastic composite being examined:

$$\langle \dot{s}_{ij} \rangle = 2M_{ijkl} \langle \dot{e}_{kl} \rangle,$$
$$M_{ijkl} = \mu_0 \{[1 + \alpha_0 B(\nu_1)] I_{ijmn} + \alpha_0 \Phi_{ijmn}(\nu_2)\}^{-1} \times$$
$$\times \{[1 - (1 - \alpha_0) B(\nu_1)] I_{mnkl} - (1 - \alpha_0) \Phi_{mnkl}(\nu_2)\} \tag{2.17}$$

(M_{ijkl} is the tensor of the macroscopic shear moduli of the medium).

We obtain the following relation for the volumetric component of the macroscopic stresses

$$\langle \sigma_{ii} \rangle = 3K_0 \langle E_{ii} \rangle. \tag{2.18}$$

The macroscopic plastic strains in the material are determined from (2.11) with allowance for (2.4), (2.7), and (2.15)

$$\mathscr{E}_{ij} = \sum_{q=1}^{N} c_q m_q^0 (\upsilon_{q1} \langle \varepsilon_{ij} \rangle_{q1} + \upsilon_{q2} \langle \varepsilon_{ij} \rangle_{q2}). \tag{2.19}$$

Equations (2.17)-(2.19) completely describe the elastoplastic state of the medium in the region of regular strain-hardening ($\nu_1 \sim \nu_2 \sim \nu_3 \neq 0$). In the elastic region, there are no plastic grains in the components ($\nu_1 = \nu_2 = 0$) and Eq. (2.17) coincides with the macroscopic Hooke's law for a stochastic mixture $\langle \dot{s}_{ij} \rangle = 2\mu_0 \langle \dot{e}_{ij} \rangle$. If the material contains only one plastic component (N − M = 1) or if all (N − M) of the components are plastic, the initial shear stresses $\tau_0^{(q)}$ (q = M + 1, ..., N) will be similar and we can assume that $\nu_{q1} = 0$, $\nu_{q2} = \delta\nu_{q2}^*$, $\nu_{q3} \sim 1$, where $\delta \ll 1$, $\nu_{q2}^* \sim 1$. Then the following is valid to within terms of the order δ

$$M_{ijkl} = \mu_0 [I_{ijkl} - \delta\Phi_{ijmn}(\nu_2)].$$

At the end of the section corresponding to regular strain-hardening of the component with the highest value of $\tau_0^{(q)}$, most of the crystallites in the plastically deformed components will be fully plastic. However, due to the plastic anisotropy of the materials (which is related to the discrete spatial arrangement of the different slip systems), there are always some grains that remain elastic even under fairly high external stresses. Nevertheless, when the number of slip systems is large (fcc and bcc crystals) the number of such grains is negligible and we can put $\nu_{q1} \simeq 1$, $\nu_{q2} = \delta\nu_{q2}^*$, $\nu_{q2}^* \sim 1$, $\nu_{q3} \simeq 0$ (q = M + 1, ..., N). In this case, the following representation is valid (to within terms of the order δ) for the tensor M_{ijkl}

$$M_{ijkl} = \mu_0 \left\{ \frac{1 - (1 - \alpha_0) B(\nu_1)}{1 + \alpha_0 B(\nu_1)} I_{ijkl} - \delta \frac{\Phi_{ijkl}(\nu_2)}{[1 + \alpha_0 B(\nu_1)]^2} \right\}.$$

Upon attainment of a state of complete plasticity, the assumptions $\nu_{q1} \simeq 1$, $\nu_{q2} \simeq 0$, $\nu_{q3} \simeq 0$ (q = M + 1, ..., N) are valid in all of the plastic components. The validity of these assumptions reduces Eq. (2.17) to the form

$$\langle \dot{s}_{ij} \rangle = 2\mu_* \langle \dot{e}_{ij} \rangle,$$
$$\mu_* = \mu_0 \left[1 - \frac{B(1)}{1 + \alpha_0 B(1)} \right], \quad B(1) = \sum_{q=M+1}^{N} c_q m_q^0 m_q^*.$$

It follows from the above relation that linear strain-hardening with the shear modulus μ_* occurs at the stage of complete plasticity in all of the plastically deformable components of the material. This modulus decreases with an increase in the concentration of plastic components in the mixture.

3. Let us examine the use of the proposed method by employing the example of the tension of a composite by the $\langle \sigma_{11} \rangle$. We have the following for the deviators of the mean stresses and strains in the composite and the components

$$[\dot{s}_{11}] = \frac{2}{3} [\dot{\sigma}_{11}], \quad [\dot{s}_{22}] = [\dot{s}_{33}] = -\frac{1}{3} [\dot{\sigma}_{11}], \quad [\dot{s}_{ij}] = 0 \quad (i \neq j),$$
$$[\dot{e}_{11}] = \frac{2}{3} (1 + \nu) [\dot{E}_{11}], \quad [\dot{e}_{22}] = [\dot{e}_{33}] = -\frac{1}{3} [\dot{E}_{11}], \quad [\dot{E}_{ij}] = 0 \quad (i \neq j), \tag{3.1}$$

where $[\,] = \langle\,\rangle, \langle\,\rangle_q$; $\nu = \nu_0, \nu_q$.

With allowance for (3.1), we find from (2.7) that

$$\dot{\eta}_{11} = \frac{2}{3} (1 - \alpha_0)(1 + \nu_0) \langle \dot{E}_{11} \rangle + \frac{\alpha_0}{3\mu_0} \langle \dot{\sigma}_{11} \rangle = (1 - \alpha_0) \mathscr{E}_{11} + \frac{\langle \dot{\sigma}_{11} \rangle}{3\mu_0},$$
$$\dot{\eta}_{22} = \dot{\eta}_{33} = -\frac{1}{2} \dot{\eta}_{11}, \quad \dot{\eta}_{ij} = 0 \quad (i \neq j). \tag{3.2}$$

Taking into account that $\Psi_{11jj}^{(q)} = 0$ by virtue of the definition of the tensor $\Psi_{ijkl}^{(q)}$, we obtain the following relation from (2.15) and (3.1)-(3.2)

$$\langle \dot{\sigma}_{11} \rangle_q = 3\mu_0 m_q^0 \{1 - (1 - \alpha_0) m_q^* [\nu_{q1} + \nu_{q2} \psi_q]\} \dot{\eta}_{11}$$
$$\left(\psi_q = \frac{3}{2} \Psi_{1111}^{(q)}, \quad q = M + 1, \ldots, N \right). \tag{3.3}$$

The strain-hardening parameters of the components of the composite $\vartheta_q = h_q(2\mu_0)^{-1}$, entering into m_q^*, should be determined from empirical tension curves obtained for each component. Assuming that $\mu_0 = \mu_q$, $\nu_{q2} = 0$ (q = M + 1, ..., N), we find from Eq. (3.3) that the below is valid for a specimen of the q-th material at the completely plastic stage

$$\dot{\sigma}_q = H_q \vartheta_q' (1 + \vartheta_q')^{-1} \dot{E}_q = H_q \vartheta_q' \dot{\mathscr{E}}_q.$$

Here, $\sigma_q = \langle \sigma_{11} \rangle_q$, $E_q = \langle E_{11} \rangle_q$, $\mathscr{E}_q = \mathscr{E}_{11(q)}$ are components of the corresponding tensors in the q-th specimen; $\vartheta_q' = h_q(2\mu_q)^{-1}$; $H_q = 2\mu_q(1 + \nu_q)$ is the elastic modulus of the material component.

In accordance with [4], for ϑ_q we find

$$\vartheta_q = m_q \vartheta_q', \quad \vartheta_q' = \omega_q (1 - \omega_q) = z_q$$
$$(\omega_q = H_{q*} H_q^{-1}, \quad z_q = H_{q**} H_q^{-1}),$$

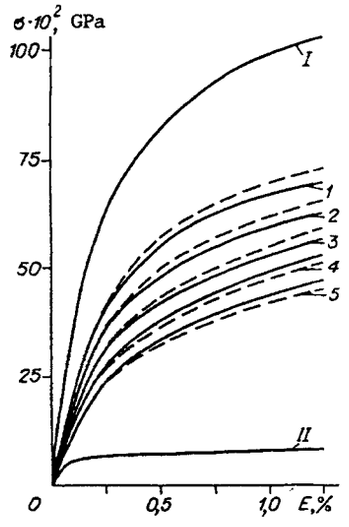

Fig. 1

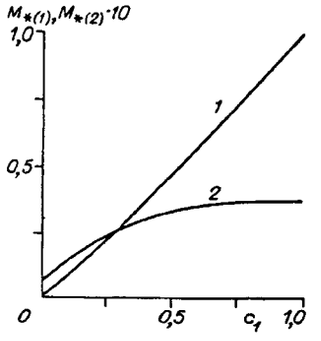

Fig. 2

where H_{q*} is the shear modulus on the experimental curve of the specimen beyond the transitional section; H_{q**} is the corresponding plastic modulus.

Equation (3.3) makes it possible to calculate the elastoplastic tension curve of a composite by the following scheme. First we use assigned values of μ_q, K_q, and c_q (q = 1, ..., N) to calculate the macroscopic elastic constants of the medium from the equations of the self-consistent averaging scheme. We also calculate the strain-hardening parameters of the plastic components ϑ_q (q = M + 1, ..., N) from the corresponding experimental curves. Then for each increment of the parameter $\dot{\eta}_{11}$ we determine the volume concentration of plastic grains v_{q1} and v_{q2} and the values of the parameters ψ_q (q = M + 1, ..., N). Equations (3.3) are used to calculate the corresponding increments of the stresses $\langle\sigma_{11}\rangle_q$ in the components. The increment of the macroscopic stress $\langle\sigma_{11}\rangle$ is found from the mixture rule (2.14), while the quantities $\langle\dot{E}_{11}\rangle$ and $\mathscr{E}_{11}$ are obtained from Eq. (3.2).

The macroscopic tension equation of the composite is determined in the following form from (2.16) with allowance for (3.2) and the condition $\Phi_{11ij} = 0$

$$\langle\dot{\sigma}_{11}\rangle = H_* \langle\dot{E}_{11}\rangle, \quad H_* = H_0 \frac{1 - (1 - \alpha_0)\,[B(v_1) + \varphi(v_2)]}{1 + \alpha_0\,[B(v_1) + \varphi(v_2)]}. \tag{3.4}$$

Here, $H_0 = 2\mu_0(1 + \nu_0)$ is the elastic modulus of the medium; $\varphi(v_2) = (3/2)\Phi_{1111}(v_2)$.

We find that $\varphi(v_2) = 0$, $H_* = H_0\{1 - B(1)\,[1 + \alpha_0 B(1)]^{-1}\}$ at the completely plastic stage, and in Eq. (3.4) we can change over from increments to the corresponding final values.

We calculated stress-strain curves for a two-component component made by impregnating a framework of sintered powdered tungsten with molten copper (Fig. 1). In accordance with the data in [3, 6, 11], the experimental tension curves of the elements of the composite conform to curves I [tungsten (q = 1)] and II [copper (q = 2)], which in turn correspond to the following values for the material constants:

$$H_1 = 413 \text{ GPa}, \quad \mu_1 = 159 \text{ GPa}, \quad \sigma_{01} = 0{,}36 \text{ GPa}, \quad \vartheta_1' = 0{,}0069,$$
$$H_2 = 123 \text{ GPa}, \quad \mu_2 = 18{,}1 \text{ GPa}, \quad \sigma_{02} = 0{,}038 \text{ GPa}, \quad \vartheta_2' = 0{,}038.$$

In keeping with (1), we also took $\tau_0^{(q)} = \sigma_{0q}/2$ (q = 1, 2), where σ_{0q} is the elastic limit of the q-th component in tension.

Curves 1-5 in Fig. 1 show the stress-strain relations for the composite with different volume contents of tungsten (c_1 = 0.666; 0.582; 0.512; 0.464; 0.412). The solid lines show the results calculated by the proposed scheme, while the dashed lines

show the experimental data [3]. In calculating the parameters v_{q1}, v_{q2}, ψ_q (q = 1, 2), we chose 91 as the number of independent orientations of the grains within the first crystallographic triangle.

Given the generally good agreement between the experimental and theoretical curves, the fact that (as can be seen in Fig. 2) the theoretical stresses are lower than the experimental stresses for $c_1 > 0.5$ and higher for $c_1 < 0.5$ is probably due to understated values of σ_{01} and ϑ_1' for tungsten and overstated values of σ_{02} and ϑ_2' for copper in [6, 11] relative to the values of the same elements in the composite examined in [3]. These components have different characteristics in the composite.

Figure 2 shows the dependence of the dimensionless plastic modulus of linear strain-hardening $M_* = \mu_*\mu_0^{-1}$ on the volume concentration of tungsten in the cases when one or both (curves 1 and 2) of the components are completely plastic.

Let us analyze the results. It follows from (3.3) that the macroscopic elastic limit of the composite is determined by the elastic limit of copper in the mixture $\langle\sigma_{11}\rangle_0 = \sigma_{02}/m_2^0$. This quantity has the following values for curves 1-5 in Fig. 1: 5.93; 5.52; 5.18; 4.97; 4.79 GPa·10^2. Copper is only partially plastic and tungsten is elastically deformed in the stage of intensive strain-hardening which follows the elastic stage (E ≃ 0.05-0.25%). Here, for a composite of constant composition, the plastic modulus decreases linearly with an increase in concentration v_{22}, while with different volume contents of the components the modulus also decreases linearly as copper concentration c_2 increases. On the section E ≃ 0.25-0.5% (see Fig. 1) (copper completely plastic and tungsten elastic), H_* is proportional to the concentration of the elastic phase (curve 1 in Fig. 2). The stage of deformation of the composite corresponding to the beginning of plastic strain of tungsten (E ≃ 0.5-1.25%) is characterized by a smooth decrease in the plastic modulus to a value corresponding to linear strain-hardening of the material, with both components being completely plastic (curve 2 in Fig. 2). At the theoretical concentrations of tungsten for curves 1-5 in Fig. 1, the plastic modulus is weakly dependent on c_1 and is close to the H_* of tungsten.

It should be noted that since the macroscopic rheological equation of the composite in the given model was obtained using a self-consistent averaging scheme, it can be employed within a fairly wide range of volume concentrations of the components and their elastic moduli — including cases in which pores and rigid phases are present [10]. The hypothesis [8] on the equality of the plastic shears in the active slip systems of a plastic grain is valid only for materials in which the number of independent slip systems in the crystals n = 5 (i.e., fcc and bcc crystals) [4]. The model presented here is valid within this region as well.

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